Benha University

Shoubra Faculty of Engineering

Photogrammetry II
Lecture 4: Camera Calibration

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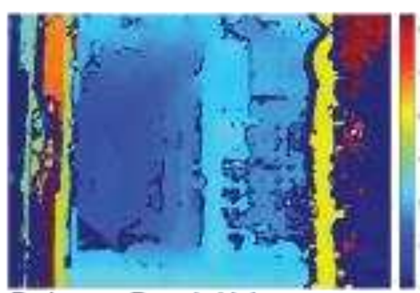
What you learn from this lecture

1. What are intrinsic parameters of the camera?
2. What is the camera matrix? (intrinsic + extrinsic)
3. How to obtain camera matrix by minimizing error
4. How to obtain intrinsic parameters from camera matrix

Remove Lens Distortion

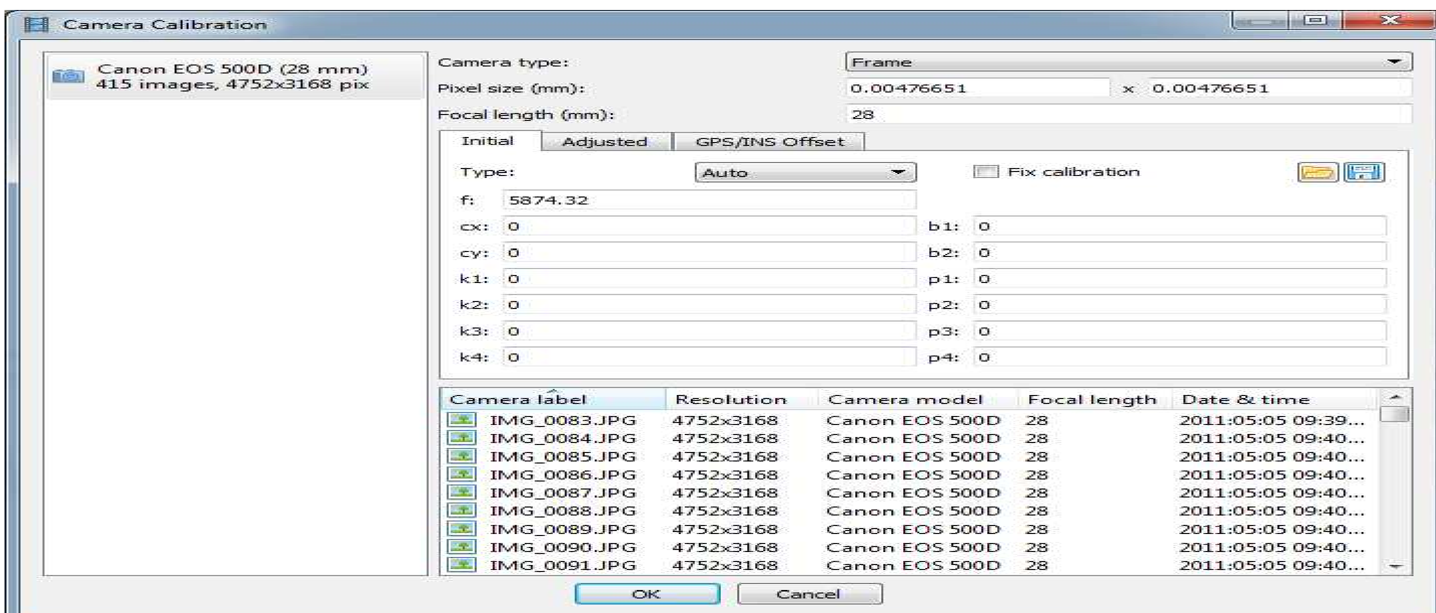

Estimate Depth Using a Stereo Camera

Measure Planar Objects


Estimate 3-D Structure from Camera Motion

The Purpose of Calibration

- Intrinsic parameters: relate the camera's coordinate to the idealized coordinate system.
- Determine the IOP of camera: Focal length (f), The principal point co-ordinates (Xo, Yo), Lens distortion parameters
- Extrinsic parameters: related the camera's coordinate to a fixed world coordinate system and specify its position and orientation in space.

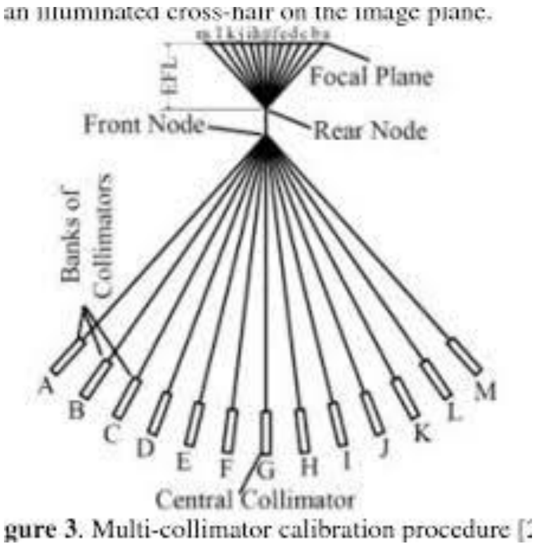


The Purpose of Calibration

Methods of Calibration

- Laboratory methods:

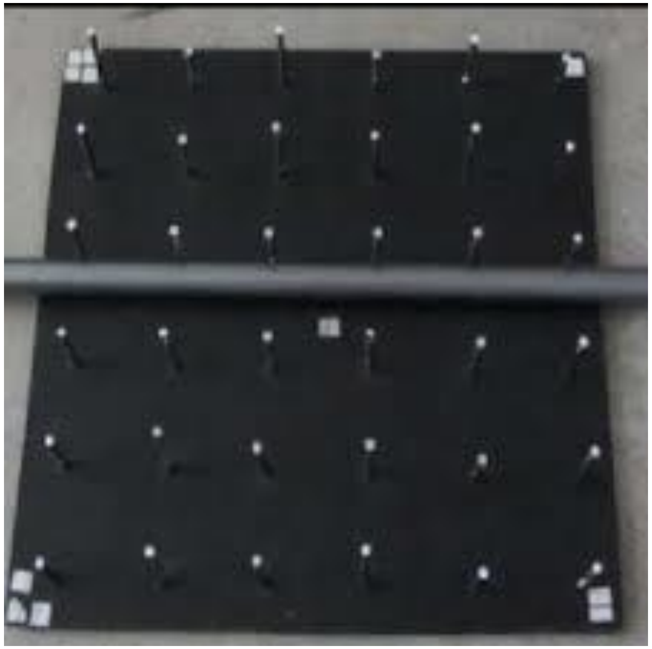
1. Goniometer
2. Multicollimator calibrator



Methods of Calibration

- Field methods:

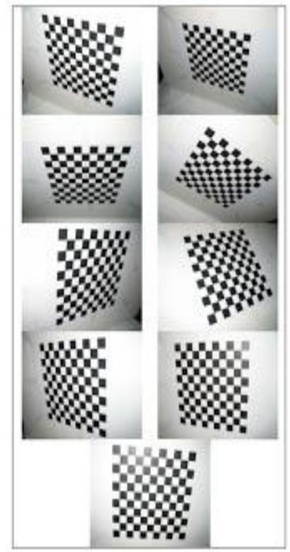
1. Stellar metohd

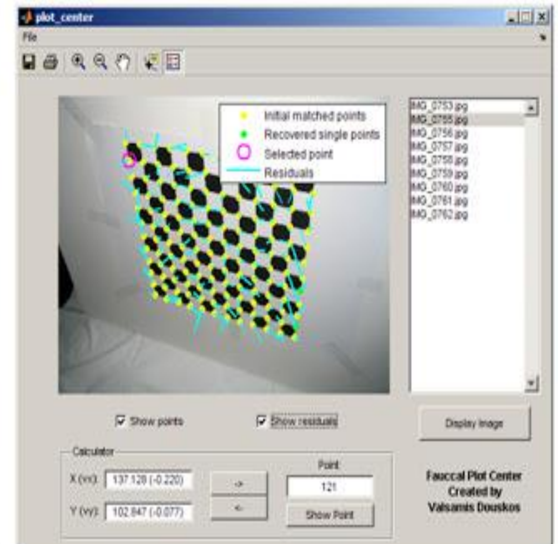


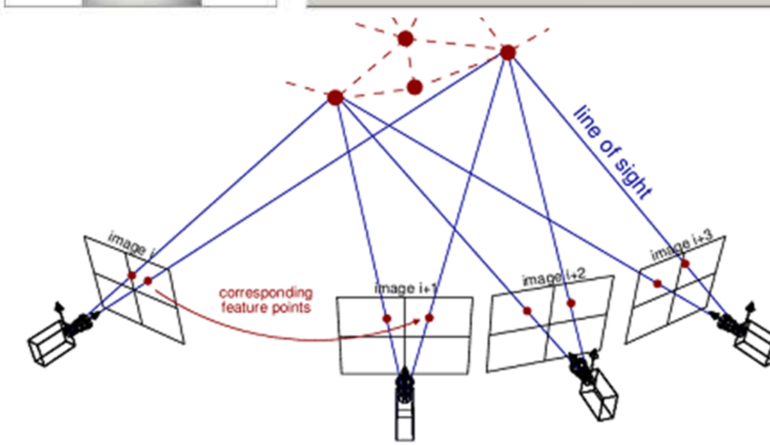
Methods of Calibration

- Calibration using 2D/3D calibration objects

- Bundle Adjustment method

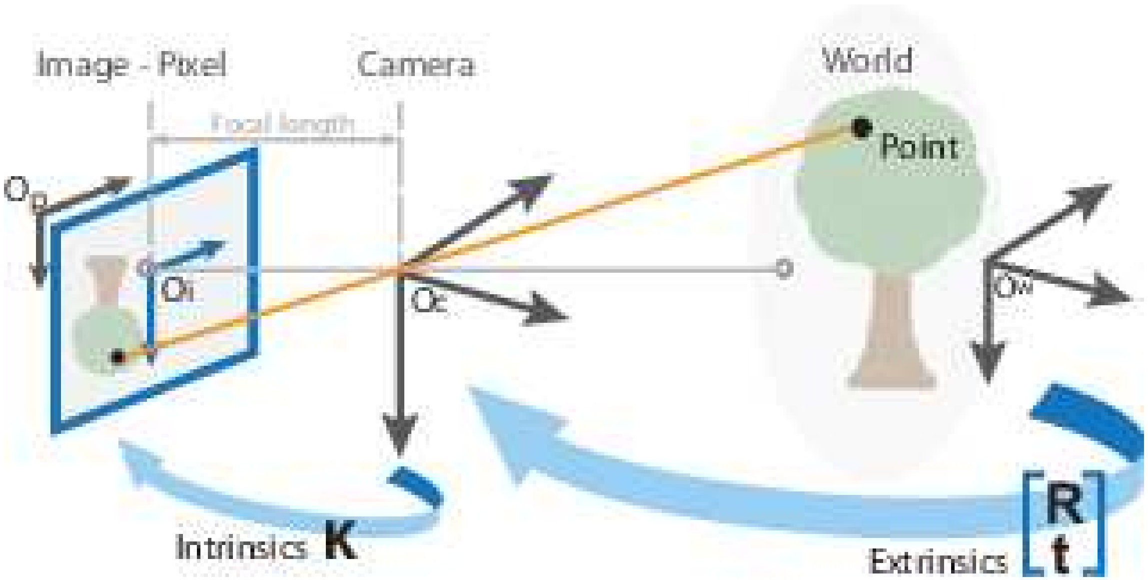






Camera Calibration using 2D/3D Objects

Camera lens perspective center, the ground point and the corresponding point on the image lies in a straight line (Collinear)



$$w\,[x\ y\ 1] = [X\ Y\ Z\ 1]\,P$$

Scale factor — Image points — World points

$$P = \begin{bmatrix} R \\ t \end{bmatrix} K$$

Camera matrix — Extrinsics: Rotation and translation — Intrinsic matrix

Camera Calibration using 2D/3D Objects

A point is expressed with several coordinate system.

3D points in world coordinate

A point $\mathbf{X}_w = (X_w, Y_w, Z_w)^\mathbf{T}$ in a world coordinate.

3D points in camera coordinate

A point $\mathbf{X}_c = (X_c, Y_c, Z_c)^\mathbf{T}$ in a camera coordinate.

2D points in image coordinate

A point $\mathbf{x} = (x, y)^\mathbf{T}$ in an image plane.

A 3×4 projection matrix $\mathbf{P}$ denotes relationship between $\mathbf{X}_w$ and $\mathbf{x}$ as

$$\mathbf{x} = \mathbf{P}\mathbf{X}_w, \tag{1}$$

$$\rightarrow s \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} p_{11} & p_{12} & p_{13} & p_{14} \\ p_{21} & p_{22} & p_{23} & p_{24} \\ p_{31} & p_{32} & p_{33} & p_{34} \end{bmatrix} \begin{bmatrix} X_w \\ Y_w \\ Z_w \\ 1 \end{bmatrix}. \tag{2}$$

Camera Calibration using 2D/3D Objects

A projection matrix can be decomposed into two components, intrinsic and extrinsic parameters, as

$$\mathbf{x} = \mathbf{P}\mathbf{X}_w = \mathbf{A}\left[\mathbf{R}|\mathbf{t}\right]\mathbf{X}_w, \tag{3}$$

$$\rightarrow \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} \alpha_x & s & x_0 \\ 0 & \alpha_y & y_0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} r_{11} & r_{12} & r_{13} & t_1 \\ r_{21} & r_{22} & r_{23} & t_2 \\ r_{31} & r_{32} & r_{33} & t_3 \end{bmatrix} \begin{bmatrix} X_w \\ Y_w \\ Z_w \\ 1 \end{bmatrix}, \tag{4}$$

where

- Intrinsic: 3×3 calibration matrix $\mathbf{A}$.
- Extrinsic: 3×3 Rotation matrix $\mathbf{R}$ and 3×1 translation vector $\mathbf{t}$.

Camera Calibration using 2D/3D Objects

Denotes transformation between $\mathbf{X}_w$ and $\mathbf{X}_c$ as

$$\mathbf{X}_c = [\mathbf{R}|\mathbf{t}]\,\mathbf{X}_w, \tag{5}$$

$$\begin{bmatrix} X_c \\ Y_c \\ Z_c \\ 1 \end{bmatrix} = \begin{bmatrix} r_{11} & r_{12} & r_{13} & t_1 \\ r_{21} & r_{22} & r_{23} & t_2 \\ r_{31} & r_{32} & r_{33} & t_3 \end{bmatrix} \begin{bmatrix} X_w \\ Y_w \\ Z_w \\ 1 \end{bmatrix}. \tag{6}$$

Camera Calibration using 2D/3D Objects

Project a 3D point $\mathbf{X}_c$ to image plane as

$$\mathbf{x} = \mathbf{A}\,[\mathbf{R}|\mathbf{t}]\,\mathbf{X}_w = \mathbf{A}\mathbf{X}_c, \tag{7}$$

$$\rightarrow \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} \alpha_x & s & x_0 \\ 0 & \alpha_y & y_0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} X_c \\ Y_c \\ Z_c \\ 1 \end{bmatrix}, \tag{8}$$

where

- α_x and α_y are focal lengths in pixel unit.
- x_0 and y_0 are image center in pixel unit.
- s is skew parameter.

Camera Calibration using 2D/3D Objects

Project a 3D point $\mathbf{X}_c$ to image plane as

$$\mathbf{x} = \mathbf{A}\left[\mathbf{R}|\mathbf{t}\right]\mathbf{X}_w = \mathbf{A}\mathbf{X}_c, \tag{7}$$

$$\rightarrow \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} \alpha_x & s & x_0 \\ 0 & \alpha_y & y_0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} X_c \\ Y_c \\ Z_c \\ 1 \end{bmatrix}, \tag{8}$$

where

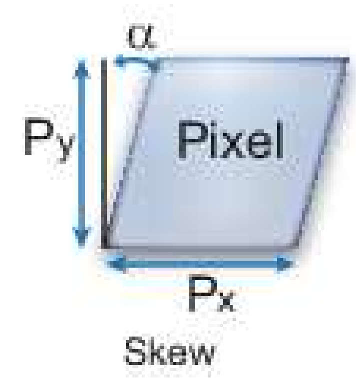
- α_x and α_y are focal lengths in pixel unit.
- x_0 and y_0 are image center in pixel unit.
- s is skew parameter.

Camera Calibration Parameters

[cx cy] — Optical center (the principal point), in pixels.

(fx, fy) — Focal length in pixels.
$fx=F/px$
$fy=F/py$
F — Focal length in world units, typically expressed in millimeters.
(px, py) — Size of the pixel in world units.

s — Skew coefficient, which is non-zero if the image axes are not perpendicular.
$s=f*\tan\alpha$



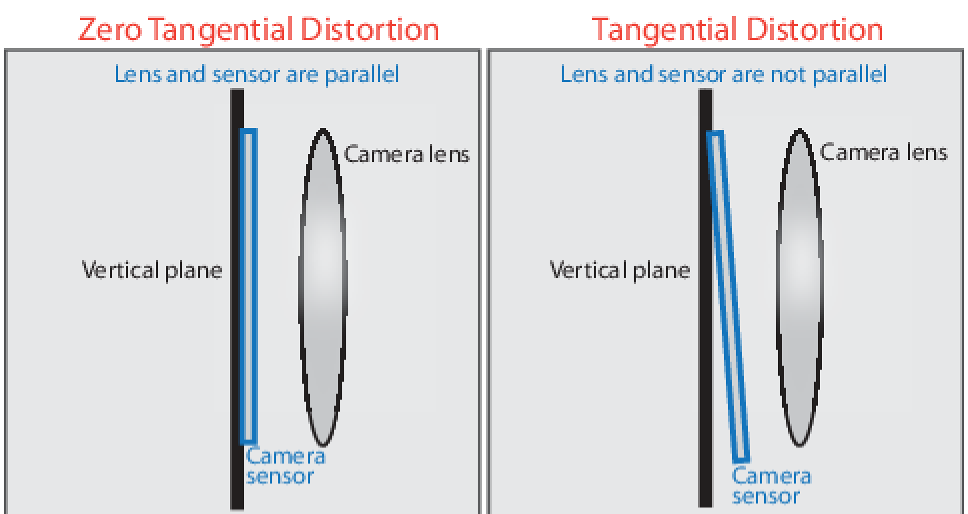
Camera Calibration Parameters

Tangential distortion occurs when the lens and the image plane are not parallel. The tangential distortion coefficients model this type of distortion.

The distorted points are denoted as (x distorted, y distorted):

X distorted = x + [2 * p1 * x * y + p2 * (r2 + 2 * x2)]

Y distorted = y + [p1 * (r2 + 2 *y2) + 2 * p2 * x * y]



Camera Calibration using 2D Checkboard

- Camera parameters requires at least 5 point pattern
- Intrinsic camera parameters estimation requires at least 3 images at different orientation
- All parameters are defined up to a unknown scale

Supplementary files:

- https://www.youtube.com/watch?v=Ou9Uj75DJX0
- https://www.youtube.com/watch?v=rgeY6FW6-Es
- https://www.youtube.com/watch?v=x6YIwoQBBxA
- Elements of Photogrammetry with Applications in GIS, Fourth Edition. Paul R. Wolf, Bon A. Dewitt, Benjamin E. Wilkinson, 2014 McGraw-Hill Education

Thanks

Dr.Eng. Hassan Mohamed